Numbers

Kronecker $\mathbb{N}$

$\mathbb{Z}$ / „Setoid" $\llbracket [(a^+,a^-)] \rrbracket = a^+ - b^-$

$\mathbb{Z}_0 = \mathbb{N} \times \mathbb{N}$ $\quad (1,2) \sim (2,3)$

$\sim\ : \mathbb{Z}_0 \to \mathbb{Z}_0 \to \mathrm{Prop}$ $\quad \sim \subseteq \mathbb{Z}_0 \times \mathbb{Z}_0$

$[\_] : \mathrm{Set} \to \mathrm{Set}$

$[\_]^s : \mathrm{Setoid} \to \mathrm{Setoid}$

$(a^+,a^-) \sim (b^+,b^-) = a^+ + b^- \equiv b^+ + a^-$

refl, sym ✓

trans $\Leftarrow \dfrac{a+b = c+b}{a=c}$ cancelation $+$

$(a^+,a^-) + (b^+,b^-) = (a^+ + b^+, a^- + b^-)$

$(a^+,a^-) \times (b^+,b^-) = (a^+b^+ + a^-b^-, a^-b^+ + a^+b^-)$

$-(a^+,a^-) = (a^-,a^+)$

---

inductive data $\mathbb{Z}^i$ =

$\quad$ neg1+ : $\mathbb{N} \to \mathbb{Z}^i$

$\quad$ zero : $\mathbb{Z}^i$

$\quad$ pos1+ : $\mathbb{N} \to \mathbb{Z}^i$

$\llbracket \mathrm{neg}\ n \rrbracket = -(n+1)$

$\llbracket \mathrm{pos}\ n \rrbracket = -(n+n)$

$(0, n+1)$

$(0,0)$

$(n+1, 0)$

neg $n = -(1+n)$

pos $n = n$

$\ulcorner \_ \urcorner : \mathbb{Z}^i \to \mathbb{Z}_0$

$nf : \mathbb{Z}_0 \to \mathbb{Z}^i$

$nf(0,0) = \mathrm{zero}$

$nf(sn, 0) = \mathrm{pos1{+}}\ n$

$nf(sn, sm) = nf(n,m)$

$nf(0, sm) = \mathrm{neg1{+}}\ m$

$\dfrac{i \sim j}{nf\ i = nf\ j}$ $\quad \ulcorner nf\ i \urcorner \sim i$ $\quad nf\ \ulcorner k \urcorner = k$

sound $\quad$ complete $\quad$ stable

$nf(a+b) = nf\ a +^i nf\ b$

$nf(a \times b) = nf\ a +^i nf\ b$

$\mathbb{Q}$ $\quad$ setoid: $(\mathbb{Z}, \mathbb{N}^+) = \mathbb{Q}_0$ $\quad (a^d, a^q) = \dfrac{a^d}{a^q}$

$\mathbb{N} \setminus \{0\}$

$(a^d, a^q) \sim (b^d, b^q)$

$= (a^d b^q \equiv b^d a^q)$

$\dfrac{a \cdot c = b \cdot c}{a = b}$

inductive $\mathbb{Z}^i = \{(a^d, a^q) \in \mathbb{Z} \times \mathbb{N}^+ \mid \gcd(a^d, a^q) = 1\}$

Σ

Field

$\mathbb{R}$ $\quad$ setoid $\quad$ Cauchy

$\mathbb{R}_0 = \{ f : \mathbb{N} \to \mathbb{Q} \mid$

$\quad \forall \varepsilon \in \mathbb{Q}^+ \exists n \in \mathbb{N}.$

$\quad |f(n+1) - f\ n| \leq \varepsilon \}$

$f \sim g = \forall \varepsilon \in \mathbb{Q}^+ \exists n \in \mathbb{N}$

$\quad \forall i \geq n.\ |f i - g i| \leq \varepsilon$

$\dfrac{\neg\neg f \sim g}{f \sim g}$